TORT LIABILITY, COURT ERRORS AND RATIONAL HINDSIGHT BIAS

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ABSTRACT

Often, courts have imperfect information regarding the injurer's investment in care. When assessing the probability that the injurer was negligent, the court can consider the fact that there was an accident, or ignore it. This Paper compares between the two alternatives. It shows that the effect of uncertainty on the injurer's incentives to invest in care depends on the effectiveness of the precaution technology. When precaution measures are less effective, uncertainty causes overdeterrence. Updating the probability of fault based on the occurrence of an accident aggravates the problem. When precaution measures are more effective, uncertainty is likely to cause underdeterrence. In these cases, updating the probability of fault based on the occurrence of an accident solves the problem of underdeterrence. Based on the analysis, the Paper offers a novel explanation to several tort law doctrines and some normative recommendations that have not been adopted yet.

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1. INTRODUCTION

Courts often have imperfect information regarding the injurer's investment in care. Knowing this, courts assess the probability that the injurer's care level fell below the standard of care, given the evidence available.¹ The court finds the injurer liable for the harm if the probability that the injurer's investment in care fell below the standard crosses a certain, predetermined threshold.² In most common-law jurisdictions this predetermined threshold is 50%, and the rule is known as the “preponderance of the evidence” (“PE”).

Behavioral law and economics scholars have suggested that courts suffer from irrational hindsight bias, and as a result, systematically over-estimate the probability that the injurer acted negligently (Kamin and Rachlinski 1995; Sunstein 2000; Jolls, Sunstein, and Thaler 1998; Labine 1996). Since courts under-estimate injurers investment in care, these scholars have suggested to reduce the standard of care, or increase the evidentiary threshold for liability, to correct for the bias. The most extreme suggestion is to give the injurer immunity from liability under certain circumstances, for example in cases where the business-judgment-rule applies (Arkes and Schipani 1994).

Rational choice theorists have argued in reply, that the so called "hindsight bias" is not an irrational cognitive bias, but rather a rational Bayesian updating of the probabilities based on additional evidence (Posner 1998; Posner 1999; Kelman, Fallas, and Folger 1998). As the argument goes – when the court has imperfect information about the injurer’s investment in care, and it knows that investment in care reduces the probability of an accident, it is perfectly rational for the court to increase its prior estimation of negligence based on the knowledge that an accident has occurred. I term this Bayesian updating process – “rational hindsight”. All else being equal, if prior to updating the court was under the assumption that errors in estimation were symmetric, after updating the distribution becomes biased against the injurer. Even though this phenomenon is perfectly rational, we should still consider whether it is warranted or

¹ I use the term "court" throughout this Paper to refer to the fact-finder. Since the argument refers to the assessment of a rational adjudicator it is irrelevant if the fact-finder is a single judge or multiple members of a jury.

² Theoretically, we can set an ad hoc threshold for each case. Kaplow (2012; 2013) suggested setting a threshold on a case-by-case basis (or by categories of cases), based on the relative costs of false positive and false negative. This suggestion has the potential to solve some of the problems raised in this Paper. However, it might create some issues regarding the costs of the legal system which are beyond the scope of this Paper.
Consider the following example:

*Example 1. Speeding Cars.* The court considers two cases. In the first, a car was detected by a speed camera moving at 66 MPH, in a 65 MPH zone. In the second case a car was involved in an accident. An expert witness has examined the glass spatter and brake marks on the street, and determined that the car was moving at a speed of 66 MPH where the standard of care was 65 MPH. Both detecting technologies are correct on average, but might still make an error in a particular reading. Furthermore, both detecting technologies are accurate to the same degree (have the same distribution of errors). Should the court determine that there is a difference in the probability of fault of the two cases?

First notice that if the detecting technologies might make an error of more than 1 MPH, the court cannot tell if both drivers were at fault with certainty. At first sight it appears that the probability that the first car was moving at a speed higher than 65 MPH is identical to the second car. However, the second car was involved in an accident. The court knows that the speed of the car affects drivers' ability to avoid a collision. Since the driver of the second car did not avoid the collision, it stands to reason that her driving speed was faster rather than slower. Thus, the court is rational in making an inference about the probability of fault from the occurrence of an accident.

However, the fact that updating the probability is rational does not necessarily mean it is warranted. From the driver's perspective, she can decide at what speed to drive the car, but cannot influence the evidence regarding that speed. If the court updates the probability of fault based on the accident, the driver knows that whatever speed she would drive at, in a case of an accident, the court would assume, on average, that she drove faster than she actually did.

That doesn't mean that updating the probability is always a bad thing. Uncertainty affects the driver's speed even when the court does not update the probability. The risk of error affects the driver in two ways – first, she knows that the evidence regarding speed are not exact, so for every speed she is driving in, there is a chance that she would escape liability. That means that uncertainty causes the driver to only partially internalize the costs created by her driving speed. All else being equal, this effect causes the driver to increase her speed beyond what is socially desirable. Second, the driver
knows that she can influence the probability of being found negligent by reducing her speed. Notice that this is a private, but not a public, benefit from reducing her speed. The motivation here is not to reduce the harm from accident, but to escape liability. All else being equal, this second effect causes the driver to reduce her speed below the social optimum. The overall effect of uncertainty would depend on which of the two effects is stronger – is the first, than uncertainty would result in under-investment in care; If the second- it would result in over-investment. Updating the probability of fault seems problematic in cases of overdeterrence – by updating the probability the court increases the drivers’ motivation to slow below the socially optimum speed. However, if uncertainty causes underdeterrence, updating the probability might be positive – by increasing the likelihood that the court would find the driver liable ex-post, she internalizes more of the risk and would reduce her speed.

The desirability of rational hindsight would depend on the combined effects of it and of uncertainty. As the next section would demonstrate, both effects hinge on the effectiveness of precaution measures, by which I mean how well investment in precautions reduces the probability of an accident. Consider the next example -

**Example 2. Oil Spill** – An oil refinery exploded, resulting in the death of several workers. The families of the deceased file a wrongful death claim against the workers’ employer – an oil company. Evidence is presented at trail regarding the safety precaution installed by the company. *Should the court consider that the refinery eventually exploded when estimating the reasonableness of the company’s investment in precaution?*

The difference between example 1 and 2 is one of magnitude. The potential harm from the explosion is so severe (including environmental harm in addition to the harm to the workers), that we might reasonably expect the oil company to implement a more effective technology to prevent harm.

By the effectiveness of the technology I do not mean a higher standard of care. Think of the prevention technology in example 1 – the speed of the car. Reducing the speed of the car reduces the probability that the car would be involved in an accident. However, it might be the case that reducing the speed is not very effective in reducing accident risk, e.g., the marginal reduction in speed is relatively expensive. In this case the residual expected harm, once the investment in care is optimal, might still be quite large in

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3 The example is based on the explosion of BP’s refinery in Texas at 2005. See, __
comparison to the investment in care. However, if precaution measures are very
effective, we would expect the injurer to invest more in care, and reduce the residual
expected harm further. When the potential harm from accidents is very severe, we
might demand a more effective prevention technology as a prerequisite to the activity.
In example 2, we might demand the oil company to show that adequate care
substantially reduces the risk of an explosion before it starts to operate.

The effectiveness of precaution measures has two implications to our inquiry. First, as
precaution measures become more effective, and residual harm gets smaller, the injurer
has less incentive to over-invest in care. That means that when precaution measures are
very effective, uncertainty is likely to cause underdeterrence,

Second, as precaution measures become more effective, the occurrence of an accident
has a stronger evidentiary implication. I.e., if the court is influenced by rational
hindsight, it is more likely to determine negligence if precaution measures are more
effective, everything else being equal.

Assume that in Example 2, care was binary (either invest or not). If the probability of an
explosion was 0.1% when the company invested properly in care, and 20% if it did not,
and the evidence showed that there is 90% chance that the company invested
adequately in care, after updating the probability the court should reach the conclusion
that there is 96% that the oil company was negligent.4

The analysis in this Paper is related to two strands of literature. The first examined the
effect of error in the determination of liability on the incentives to invest in care, as in
(Craswell and Calfee 1986; Shavell 1987; Landes and Posner 1987; Miceli 1997; Dari-
Mattiacci 2004; Cooter and Ulen 2012). These papers identified the effects of
uncertainty regarding the injurer’s care level or the standard of care, on the injurer’s
incentives to invest in care. However, the courts in these papers were modeled as if they
are unaware that they possess imperfect information and might err as a result. This
assumption is odd, considering the emphasis on the standard of proof in legal
proceedings. However, by assuming that only the injurer knows the distribution of court
errors, allows the model to include biased error distribution, as in (Craswell and Calfee
1986). In this Paper I assume that both the injurer and the court know the distribution

4 This is a simple Bayesian updating of the probability – the prior odds are 9/1, and that
estimation should be multiplied by 1/200 (the odds of an accident, given adequate and
inadequate investment in care). The result is odds of 9/200, or probability of 96%.
of error when assessing the injurer’s behavior, and that the court calculates, based on
the distribution, the probability that the injurer was negligent.

The second strand of literature examined different evidentiary threshold that courts can
use to determine liability when they possess imperfect information regarding care.
Johnston (1987) and Fluet (2010) argued that uncertainty usually causes
overdeterrence, and that courts should response by reducing the standard of care or
increasing the evidentiary threshold for liability. In Johnston (1987) the model
incorporates biased decisions, but fails to explain why court’s decision might be biased if
courts are aware of the distribution of error. Demougin and Fluet (2006; 2008)
determined that uncertainty can result only in underdeterrence, and concluded that PE
is the best evidentiary threshold under uncertainty. However, Previous papers did not
consider the possibility of updating the court’s assessment based on the occurrence of
an accident.5 Furthermore, previous literature did not consider the efficiency of
precaution measures as a factor in either the effects of uncertainty or the distribution of
court errors.

The paper will continue as follows – Part 2 of the Paper develops a model of uncertainty
regarding the injurer’s care level, in a unilateral accident. I first examine the effects of
uncertainty without rational hindsight, and then add the possibility of updating the
court’s assessment based on the occurrence of an accident. Part 3 of the Paper argues
that the analysis can explain several known tort law doctrines, for example, the doctrine
of abnormally dangerous activities places strict liability on the injurer, regardless of the
reasonableness of precaution measures.6 “Abnormally dangerous activity” is defined as
an activity that “creates a foreseeable and highly significant risk of physical harm even
when reasonable care is exercised by all actors “.7 In the terms of this paper, the
definition includes two condition – that the size of potential harm is significant, and that
precaution measures are less effective – when precaution measures are less effective the
residual expected harm when reasonable care is exercised remines high. As the model
proves, under these conditions injurers are likely to over-invest in care. A regime of

5 Ben-Shahar (1995) has considered the effect of judging in hindsight, but limited his analysis to
cases in which the court has new information at trail regarding the standard, that was not
previously available to the injurer.

6 See Restat 3d of Torts: Liability for Physical and Emotional Harm, § 20 (3rd 2010). The clause
states that “An actor who carries on an abnormally dangerous activity is subject to strict liability
for physical harm resulting from the activity.”

7 Id.
strict liability solves the problem of overdeterrence. This suggest, somewhat counter-intuitively, that a regime of strict liability for abnormally dangerous activity is aimed to reduce the injurer's investment in care, and not increase it.

2. A Model of Court Errors

We start with the standard unilateral accident model, with two risk-neutral parties: an injurer and a victim. Only the injurer can invest in precaution measures that reduce the probability of an accident. If an accident occurs only the victim suffers harm. The interaction between the injurer and the victim is governed by a negligence regime, under which if an accident occurs, and negligence is established, the injurer must fully compensate the victim, even if the accident would have occurred had the injurer been non-negligent.

Let $x$ denote the injurer’s level of care, where $x > 0$. The probability of an accident, $p(x)$, is assumed to be twice continuously differentiable, where $p'(x) < 0$ and $p''(x) > 0$. $H$ denotes the harm to the victim in a case of an accident ($H > 0$). Damages are assumed to be fully compensatory (i.e. damages equal $H$).

The social cost of accidents is defined as the sum of the injurer’s investment in precaution and the victim’s expected harm due to accidents, which equals $x + p(x)H$. The socially optimal investment in precaution, denoted as $x^*$, minimizes the social costs of accidents. Solving for the first-order condition, we obtain $p'(x^*)H = -1$. The second-order condition yields $p''(x^*)H > 0$ for all $x$, showing $x^*$ is a local minimum.

We assume that $x^*$ is positive and known to both the injurer and the court. The standard of care is set at $x^*$, so the court finds the Injurer negligent if $x < x^*$.

Assume that courts cannot observe the investment of care directly, but possess a technology that detects the injurer’s investment in care $x$ with some noise or error. Let $\varepsilon$ denote the error in the assessment, and let $\bar{x}$ denote the observed level of care, so $\bar{x} = x + \varepsilon$.

The probability density of $\varepsilon$, $f(\varepsilon)$, and the cumulative distribution function of $\varepsilon$, $F(\varepsilon)$, are known to the injurer and to the court.\(^8\) We assume that $\varepsilon$ is independent from the level

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\(^8\) This assumption is not obvious. The injurer knows the actual level of care, and observes the court's decision. It could be argued that the injurer is in a better position to know the court's error function than the court. However, that requires the injurer to observe several decisions about other injurers and to observe the actual investment in care of those other injurers.
of care $x$. In other words, when the court observes $\bar{x}$ it believes that it comes from the same distribution function $f$ regardless of the size of $\bar{x}$.

For example, a speed camera measures the speed of a moving vehicle. If the speed camera is calibrated, the speed it detects is correct on average. However, it might still err on any particular reading. Knowing that the speed camera might err, and the distribution of error, the court can calculate the probability that a car’s speed crossed the speed limit, given the speed detected by the speed camera.

Under these assumptions, from the perceptive of the court, the probability that the injurer took less precaution than due care is given by $F(x^* - \bar{x})$. To see why, notice that $P(x \leq x^* | \bar{x}) = P(\bar{x} + \epsilon \leq x^*) = P(\epsilon \leq x^* - \bar{x}) = F(x^* - \bar{x})$. From the injurer’s perspective, if she invests $x$ in precaution than the probability that the court will observed a specific $\bar{x}$ is given by $f(\bar{x} - x)$.

**Proposition 1.** If the court knows the distribution of $\epsilon$, then there will be no bias in determining liability based on the detection technology, regardless of the distribution of error.

**Proof.** The distribution function of the error is biased, if the error made by the detection technology systematically underestimate or overestimate the level of precaution taken by the injurer. Since the probability density function $f(\epsilon)$ is known to the court, it will correct for the bias when estimating the probability of negligence based on the observed investment of care.

If we look back at the example of the speed camera, assume that the error distribution is uniform between the actual speed and 10 MPH above the actual speed. Under these assumptions, the expected reading of the speed camera will be 5 MPH above the actual speed, which means that the error distribution is biased. However, the court’s estimation would not be biased – for example if the speed camera recorded a speed of 53 MPH, in a 50 MPH zone, the court, knowing the distribution of error, would estimate that the probability that the speed was higher than 50 MPH is only 30%.

When the court knows the distribution of the errors, and assess the probability of fault based on that distribution, it might err in a particular instance, but would be correct on

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Assuming injurers are not repeated players, it is likely that injurers would have better information then the courts regarding the distribution of court errors.

9 This is a common assumption in the literature. See, e.g. (Shavell 1987).
average. Accordingly, because biased functions will not cause biased decisions, we can assume for simplicity that \( f \) is a symmetric function, which means that the mean and the median of the distribution are zero. Since the median error is zero, it follows that \( F(0) = 0.5 \).

**Proposition 2.** If the court applies rational hindsight it would systematically under-evaluate the injurer’s investment in care.

**Proof.** For every observed level of care \( \bar{x} \), the court knows that the probability that the injurer invested a specific amount in care is given by \( f(x - \bar{x}) \). Under rational hindsight the court can update the probability by the likelihood of an accident occurring for every level of care, using Bayesian updating. The updated probability of any specific investment \( x \), given the observed investment \( \bar{x} \), and the occurrence of an accident (which I denote as \( a \)) is given by-

\[
(1) \ P(x|\bar{x}, a) = \frac{f(x-\bar{x})p(x)}{\int_{-\infty}^{\infty} f(x-\bar{x})p(x)dx}.
\]

Since \( p(x) \) is decreasing in \( x \), the updated estimation gives more weight to \( x \) values that are below \( \bar{x} \), and less weight to \( x \) values above \( \bar{x} \). The updated probability that the injurer was negligent is the sum of the probabilities of all possible investments in care, up to the standard. This is given by –

\[
(2) P(x < x^*|\bar{x}, a) = \frac{\int_{-\infty}^{x^*} f(x-\bar{x})p(x) dx}{\int_{-\infty}^{\infty} f(x-\bar{x})p(x)dx}
\]

It follows from exp. 1 that the value of exp. 2 is higher than the probability of negligence before updating, \( F(x^* - \bar{x}) \), for every level of \( \bar{x} \). By updating the distribution, the court gives more weight to lower levels of care, which would result in a higher probability of negligence.

Following this structure, we can now compare the effects of uncertainty on the incentives to invest in care, first assuming symmetric distribution of errors (i.e. that courts refrain from rational hindsight) and then assuming asymmetric distribution of uncertainties.

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\(^{10}\) To be more precise, the probability of negligence after updating is never lower, but can be equal. When the prior probability is 1 or 0, updating it would not cause a change. That means that if the court knew before updating that the injurer was negligent, or that she was not negligent, with certainty, taking into account the fact that an accident occurred would not influence the court’s previous estimation.
errors by updating the distribution given the occurrence of an accident.

2.1. Incentives Under Symmetric Distribution of Errors

Under PE, the court imposes full liability if and only if the probability of fault is greater than 50%. Recall that from the court's perspective, the probability that the injurer was negligent is \( F(x^* - \bar{x}) \). Thus, under PE the court will find the injurer negligent whenever \( F(x^* - \bar{x}) \geq 0.5 \)

**Proposition 3.** Under PE, the court should impose liability whenever the observed level of precaution is lower than the standard of care \( (\bar{x} < x^*) \).

**Proof.** Recall that the probability, from the court's perspective, that the injurer was negligent is given by \( F(x^* - \bar{x}) \). Since \( F(0) = 0.5 \), and \( F \) is monotonically increasing, courts should impose liability whenever \( \bar{x} < x^* \).

**Proposition 4.** Under a negligence regime, PE may cause either under-deterrence or over-deterrence.

**Proof.** The probability that the court will observe a particular level of investment in care, \( \bar{x} \), is \( f(\bar{x} - x) \). From Proposition 3, the injurer knows that she will have to bare the entire harm if \( (\bar{x} < x^*) \). Thus, from the injurer's perspective the probability that the court, applying PE, will find her negligent if she invested \( x \) in care is:

\[
\int_{-\infty}^{x^*} f(\bar{x} - x)d\bar{x} = F(x^* - x)
\]

It follows that under the PE, the injurer's cost function is:

\[
C_{PE} = x + F(x^* - x)p(x)H
\]

Solving for the fist-order condition, we obtain:

\[
1 = f(x^* - x)p(x)H - F(x^* - x)p'(x)H
\]

The LHS of Exp. 5, 1, is the marginal cost of care. The RHS of Exp.5 is made of two components: The first, is the marginal reduction in expected liability, i.e. the reduction in the probability that the court would find the injurer liable, times the expected liability that is now avoided. All else being equal, this effect of uncertainty induces the injurer to invest more in precaution. The second is the reduction in accident cost from the added level of care, multiplied by the probability that the court will impose liability. Since the
injurer internalizes the excepted harm from accidents only if she is found liable, she internalizes only a part of the marginal reduction in accident costs. This effect causes the injurer to invest to little in precaution. We will refer to the first effect of uncertainty as the "Private benefit of added precaution", since it describes a private, and not public, benefit to the injurer created from additional unit of precaution. We will refer to the second effect as 'Partial internalization of added precaution', since the injurer internalizes only part of the reduction in the social costs of accidents. Let \( x_s \) denote the \( x \) value that satisfices Exp. 5.

Similar results were obtained by Craswell and Calfee (1986a), Landes and Posner (1986), Shavell (1987), Miceli (1997), and Dari-Mattiacci (2005). However, while these models have (implicitly) assumed that even though courts might err, they believe that their decision is accurate, the above analysis shows that the result holds, even if courts do not hold this belief, as long as they apply PE.

**Proposition 5.** Under a symmetric distribution of errors, the effect of uncertainty will be determined by the variance of the distribution and the effectiveness of precaution measures.

To measure the effectiveness of investment in precautions, we assume that \( p(x) = x^{-\alpha} \). \( \alpha \) is the 'effectiveness parameter'. As the value of \( \alpha \) increases, investments in precaution reduce the probability of an accident more rapidly. For example, if \( \alpha = 2 \) an increase in \( x \) from 1 to 2 will reduce the probability of an accident from 1 to \( \frac{1}{4} \). However, if \( \alpha = 10 \) an increase in \( x \) from 1 to 2 will reduce the probability of an accident from 1 to \( \frac{1}{1024} \). We assume that \( \alpha \geq 1 \). Notice that under these assumptions, \( p'(x)H = -\frac{\alpha H}{x^{1+\alpha}} \) which means

\[
x^* = (\alpha H)^{1+\alpha}, \text{ and } p(x^*)H = \frac{\mu^{1+\alpha}}{\alpha^{1+\alpha}}.
\]

To further explore the effects of uncertainty, we will assume that \( \epsilon \) is distributed uniformly around 0. Parameter \( \sigma \) will be used to measure the extent of uncertainty. To connect the size of uncertainty with the model, we will set the area of the uniform distribution as a multiplier of the standard of care, \( x^* \), so \( \epsilon \sim U(-\sigma x^*, \sigma x^*) \), and \( 0 \geq \sigma \geq 1 \).

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\( ^{11} \) If we would allow for \( \sigma > 1 \), the court might observe negative levels of care even if the injurer overinvests, which contradicts the assumptions previously made about the possible levels of \( x \).
We can now plug the distribution of error and the probability of an accident to Exp. 4:

\[
(6) \quad C_{PE} = x + \frac{x + x^{-\alpha H}}{2\sigma x^*} x^{-\alpha} H \quad \frac{x^* - \sigma x^* > x}{x^* + \sigma x^* \geq x \geq x^* - \sigma x^*} \quad \frac{x > x^* + \sigma x^*}{x \geq x^* - \sigma x^*} \]

We can determine the effect of uncertainty by finding the solution for the first-order condition of Exp. 6, and comparing it to the efficient level of care \(x^*\).

\[
(7) \quad 1 = \frac{\alpha H}{x^* \sigma x^*} + \frac{(x^* - x + \sigma x^*)}{x^* \sigma x^*} \quad \frac{x^* - \sigma x^* > x}{x^* + \sigma x^* \geq x \geq x^* - \sigma x^*} \quad \frac{x > x^* + \sigma x^*}{x \geq x^* - \sigma x^*} \]

It is easy to see that \(x_s\) must fall in the interval between \(x^* + \sigma x^*\) and \(x^* - \sigma x^*\). If \(x_s\) is smaller than \(x^* - \sigma x^*\), the RHS of Exp. 7 equals \(-p'(x_s)H\). We know that the expression equals 1 when \(x = x^*\), which means that it must be higher than 1 for any \(x < x^*\). This result is very intuitive – when the injurer’s investment in care is low enough, she knows the court will find her negligent with certainty, and is induced to increase her investment in care. For \(x > x^* + \sigma x^*\), the RHS of Exp. 7 is 0. The intuition behind the result is straightforward – when the injurer invested enough in precautions she knows that the court would never find her negligent, and investing more in precaution has no benefit.

**Lemma 1.** Under PE the injurer might over-invest in care in such way that she is never found liable by the court. The instances in which this would happen depend on the magnitude of uncertainty, and the effectiveness of investment in precaution.

**Proof.** Notice that Exp. 6 is a piecewise continuously differentiable function, so even though it is continuous, and must have a local minimum, it might not satisfy the first-order condition for all values of \(x\). We can find all the possible solutions by examining the minimal and maximal values of the middle interval in Exp. 7.

For \(x = x^* - \sigma x^*\), the RHS of Exp. 7 is \(\frac{aH}{(x^* - \sigma x^*)^{\alpha + 1}}\). Since \(\frac{aH}{x^*^{\alpha + 1}} = 1\), it must be that \(\frac{aH}{(x^* - \sigma x^*)^{\alpha + 1}} > 1\), which means that at the minimal value of the interval, the marginal cost of care is always lower than the marginal benefit. It follows that so \(x_s\) must be larger than \(x^* - \sigma x^*\).

For \(x = x^* + \sigma x^*\), the RHS of Exp. 7 is \(\frac{H}{2\sigma x^* (x^* + \sigma x^*)^{\alpha}}\). The injurer will invest \(x^* + \sigma x^*\) in
care (Maximal investment) whenever \(2\alpha\sigma(1 + \sigma)^{\alpha} \leq 1\). For example, if \(\alpha = 1\) the injurer will invest in care the maximal investment if \(\sigma < \frac{\sqrt{3} - 1}{2} \approx 0.366\). If, however, \(\alpha = 2\) the injurer will invest in care the maximal investment if \(\sigma < 0.179652\).\(^{12}\)

Fig. 1 shows the cases in which the injurer would over-invest in care up to the limit of the distribution, i.e. up to \(x^* + \sigma x^*\). The X axes represent the effectiveness of investment in care (captured by \(\alpha\)), and the Y axes represent the extent of uncertainty, in terms of percentage of the standard of care (\(\sigma\)).

As can be seen in Fig.1, when the distribution of error of the detection technology is low enough, that injurer would always invest in a way that the court will never find her liable. Similarly, when the error is bigger than 40% of the standard of care, the injurer would invest less in care, allowing for the possibility that court will find some injurers liable.

**Lemma 2.** Uncertainty causes under-deterrence when precaution measures are more effective and uncertainty levels are high.

**Proof.** For \(x = x^*\), \(\frac{\alpha H}{x^* + \sigma} = 1\).

Similarly, from Exp.7 when \(x = x_b\), \(\frac{H}{2\sigma x^* x^2} + \frac{(x^*-x+\sigma x^*)}{2\sigma x^* x^2+1} \alpha H = 1\). So, it must be that -

\(^{12}\)The number is the positive rational root to the polynomial \(4\sigma^3 + 8\sigma^2 + 4\sigma - 1 = 0\).
\[
\frac{H}{2\sigma x^a} + \frac{(x^* - x_b + \sigma x^*)}{2\sigma x^a} = \frac{aH}{x_b + a}
\]

Under this condition, \(x^* = x_b\) when \(\sigma = \frac{1}{a}\)

The result shows the connection between deterrence and uncertainty – we have learned from Exp.5 that uncertainty creates two effects – the Private benefit of added precaution that induces overinvestment, and the partial internalization of added precaution that induces underinvestment. However, the magnitude of uncertainty influences the two effects differently. The private benefit of added precaution \(\left(\frac{H}{2\sigma x^a} = \text{exp. 7}\right)\), gets smaller as the level of uncertainty increases (since \(\sigma\) appears only in the denominator).

The intuition is clear – when the injurer faces high levels of uncertainty, she knows that the probability that she will bear the costs of the accident depend more on chance, and less on her investment in care. In such a case, investing more in precaution creates small private benefit. The partial internalization effect, on the other hand, is determined by the cumulative distribution function, which is less sensitive to small changes in the level of uncertainty. That means that for values of \(\sigma\) lower than \(\frac{1}{a}\), uncertainty causes overdeterrence, and for values of \(\sigma\) larger than \(\frac{1}{a}\), uncertainty causes underdeterrence.

Fig.2 divides the effects of uncertainty, as a function of the ratio between the level of uncertainty and the efficiency of investment in precaution, into three areas (The X and Y axes are the same as in Figure 1). Area 1 is the same area presented in Fig.1, in which the injurer insets the maximal amount in care. In Area 2, the Injurer overinvests in care, but not to the maximal amount. In Area 3 the injurer underinvests in care.

Fig.3 offers two interesting insights – First, when precaution measures are not very effective, uncertainty can create only over-deterrence. This might explain why several papers have suggested that uncertainty only causes overdeterrence – if the models did not control for the relative effectiveness of precaution measures, the results might stem from the specific function that described the probability of an accident. Second, it illustrates that as precautions measures become more effective there is a greater chance that uncertainty would cause underdeterrence. The intuition behind the result is that as precaution measures become more effective, the expected harm from the accident decreases. That means that the private benefit from added precaution, i.e. the benefit from escaping liability, gets smaller.
2.2. Incentives Under Biased Distribution of Errors

**Proposition 6.** If the court updates the likelihood of negligence given the occurrence of an accident, the distribution becomes biased, and the effect gets stronger with the effectiveness of precaution measures.

**Proof.** We have defined in Exp.2 the updated probability that the injurer was negligent, given the occurrence of an accident. We can measure how the effectiveness of precaution measures influences the court’s estimation, by plugging the distribution of error and the probability of accidents into Exp.2-

\[
P(x < x^*|\bar{x}, \alpha) = \begin{cases} 
0 & \quad \bar{x} > x^* + \sigma \cdot x^* \\
\frac{\int_{\bar{x}-\sigma \cdot x^*}^{x^*} f(x - \bar{x})p(x) \, dx}{\int_{\bar{x}-\sigma \cdot x^*}^{\infty} f(x - \bar{x})p(x) \, dx} & \quad \sigma \cdot x^* > \bar{x} - x^* > -\sigma \cdot x^* \\
1 & \quad \bar{x} < x^* - \sigma \cdot x^*
\end{cases}
\]

Under PE, the court will find the injurer liable for the harm whenever the probability of negligence is higher than \(\frac{1}{2}\). The \(\bar{x}\) value that satisfies the condition is given by

\[
\ln(\bar{x} - \sigma \cdot x^*) - 2\ln(x^*) + \ln(\bar{x} + \sigma \cdot x^*) > 0 \Rightarrow \bar{x} < x^*\sqrt{1 + \sigma^2}.
\]

Since the value under the root is bigger than 1 for any \(\sigma > 0\), when the court updates the distribution it is more likely to find the injurer negligent.

\[\text{Exp. 10 is only true for } \alpha > 1. \text{ When } \alpha = 1, \text{ the expression equals } -\frac{\ln(\bar{x} - \sigma \cdot x^*) - \ln(x^*)}{\ln(\bar{x} - \sigma \cdot x^*) - \ln(\bar{x} + \sigma \cdot x^*)} > \frac{1}{2} \Rightarrow \ln(\bar{x} - \sigma \cdot x^*) - 2\ln(x^*) + \ln(\bar{x} + \sigma \cdot x^*) > 0 \Rightarrow \bar{x} < x^*\sqrt{1 + \sigma^2}. \text{ That means that when } \alpha = 1, \text{ the court will find the injurer negligent whenever the observed level of care is lower than the standard of care times } \sqrt{1 + \sigma^2}. \text{ Since the value under the root is bigger than 1 for any } \sigma > 0, \text{ when the court updates the distribution it is more likely to find the injurer negligent.} \]
\[
\frac{(\bar{x} - \sigma \times x^*)^{1-\alpha} - x^*^{1-\alpha}}{(\bar{x} - \sigma \times x^*)^{1-\alpha} - (\bar{x} + \sigma \times x^*)^{1-\alpha}} > \frac{1}{2} \Rightarrow (\bar{x} - \sigma \times x^*)^{1-\alpha} + (\bar{x} + \sigma \times x^*)^{1-\alpha} > 2x^*^{1-\alpha}
\]

Fig. 3 shows the updated probability that the injurer was negligent, based on observed level of care, for varying effectiveness degrees of precaution measures. The first curve from the left shows the court's estimation of fault, based on the observed level of care, assuming the court refrains from rational hindsight. As you can see, when the observed level of care is equal to the standard, the probability of negligence is exactly 50%, so the court will place no liability if the observed level of care is above the standard. The same is not true when the court updates its estimation.

The second curve (bold) shows the court's estimation when precaution measures are least effective \((\alpha = 1)\). Notice that when the court observes the standard it assumes that there is more than 50% that the injurer invested less than the standard, and place liability. Specifically, when \(\alpha = 1\) the injurer will be found liable whenever the observed investment in care is lower than \(x^* \sqrt{1 + \sigma^2}\).

The third curve (dashed) shows how updating affects the court's decisions when \(\alpha = 2\). In this case, the injurer will be found liable whenever the observed level of care falls below \(\frac{x^* (1 + \sqrt{1 + 4 \sigma^2})}{2}\). Notice that \(x^* \sqrt{1 + \sigma^2} < \frac{x^* (1 + \sqrt{1 + 4 \sigma^2})}{2}\), which makes sense – if precaution measures are more effective the court can make a stronger inference from the occurrence of an accident.

The forth curve (dotted) shows how updating affects the court's decision making when precaution measures are extremely effective. For simplicity, I used the function \(p(x) = e^{-x}\), instead of setting high value for the \(\alpha\) parameter. When investment of \(x\) in precaution reduces the probability of an accident by a factor of \(e^{-x}\), \(p'(x)H = -He^{-x}; x^* = \ln(H); \) and \(p(x^*)H = 1\). As you can see, when precaution measures are extremely effective, the expected harm from an accident, when investing optimally in precautions, is always low no matter how massive might be the harm if an accident occurs. However, when investment in precaution is so effective, the court can make a very strong inference from an accident occurring, making the injurer liable whenever there is even a slight prior probability that she was negligent.

\[\text{\textsuperscript{14}}\text{See Supra note 13}\]

\[\text{\textsuperscript{15}}\text{The probability function is different from the one used previously in the sense that its effectiveness depends of the amount of harm in case of an accident, since the residual harm under optimal investment is always 1. For Fig.3 I assumed that the standard of care was 10, which means that } H = e^{10}.\]
Proposition 7. When uncertainty causes overdeterrence, updating the probability of fault based on rational hindsight aggravates the problem.

Proof. We know from Exp. 8 that for \( \alpha = 1 \), uncertainty causes only over-deterrence. That gives us a way to examine the effects of biased distributions on the injurer’s incentives to invest in care. We know that the injurer will be found liable whenever the observed level of care falls below \( x^* \sqrt{1 + \sigma^2} \). The injurer’s updated cost function is:

\[
C_{PE}^{biased, \alpha=1} = x + \frac{(\sqrt{H(\sqrt{1 + \sigma^2} + \sigma)} - x)^2}{2 \sigma \sqrt{H}} x^{-1} H \quad \text{for}\quad \sigma \sqrt{H} \geq x - \sqrt{H} \sqrt{1 + \sigma^2} \geq -\sigma \sqrt{H} \quad \text{and}\quad x - \sqrt{H} \sqrt{1 + \sigma^2} > \sigma \sqrt{H}.
\]

When the court’s estimation is biased, the injurer’s cost function is changed in two ways. First, the boundaries of the regions change since the injurer knows she has to invest more in order to escape liability with certainty. Second, the cost function in the middle region has changed as well, since the injurer expects to be liable more often for every level of precaution. Solving for the first order condition, for the middle region, we obtain:

\[
1 = \frac{H(\sigma + \sqrt{1 + \sigma^2})}{2x^2 \sigma}
\]

It follows from Exp. 12 that the injurer would invest in care would equal \( \sqrt{H(\sigma + \sqrt{1 + \sigma^2})} \). However, we know from Exp. 11 that the injurer can avoid liability completely by investing \( \sqrt{H(\sqrt{1 + \sigma^2} + \sigma)} \) in care. Accordingly, the injurer’s investment
in care \((x_b)\) would be the minimum of the two.

\[
x_b = \min \left( \sqrt{\frac{H(\sigma + \sqrt{1 + \sigma^2})}{2\sigma}}, \sqrt{H(\sqrt{1 + \sigma^2} + \sigma)} \right)
\]

The second value is lower whenever \(\sigma < \frac{\sqrt{3} - 1}{\sqrt{8}}\), which is the condition for maximal deterrence under the biased distribution.\(^{16}\) For cases of maximal deterrence, biased distribution causes the injurer to invest more in care, since the maximal value of the range of uncertainty for the biased distribution \(\left(\sqrt{H(\sqrt{1 + \sigma^2} + \sigma)}\right)\) is higher than the value for the unbiased distribution \(\left(\sqrt{H(1 + \sigma)}\right)\).

Biased distributions cause the injurer to invest more in care (relative to unbiased distributions) even when the injurer invests less than the maximum amount. According to Exp. 7, the injurer’s investment in care when \(\alpha = 1\) is equal to \(\sqrt{\frac{H(1 + \sigma)}{2\sigma}}\). For biased distribution, the investment amount equals \(\sqrt{\frac{H(\sigma + \sqrt{1 + \sigma^2})}{2\sigma}}\). It is easy to see that the second value is bigger than the first, regardless the size of \(\sigma\).

**Proposition 8.** When precaution technology is very effective, and thus uncertainty causes underdeterrence, updating the probability of negligence solves the problem of underdeterrence.

**Proof.** When the probability of an accident equals \(e^{-x}\), and the distribution of error is symmetric, the injurer's investment in care \((x_b)\) equals:

\[
x_b = 1 + \ln(H^{1+\sigma}) - W\left(2eH^\sigma \ln(H^\sigma)\right)
\]

where \(W\) stands for the Lambert \(W\)-Function. The Value is lower than the standard of care, \(\ln(H)\), whenever \(\sigma < \frac{1}{\ln(H)}\). Since the standard of care is set at \(\ln(H)\), it means that uncertainty causes underdeterrence whenever the absolute size of \(\sigma x^*\) is smaller than 1 (which is also the residual expected harm). It allows us to examine how updating the

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\(^{16}\) Recall that when \(\alpha = 1\), maximal deterrence occurred whenever \(\sigma < \frac{\sqrt{3} - 1}{2}\). So, for values of \(\sigma\) in the (small) region \(\frac{\sqrt{3} - 1}{2} < \sigma < \frac{\sqrt{1}}{\sqrt{8}}\), the injurer would invest the maximal amount when the errors are unbiased and would not do so when the distribution is biased. However, considering that the maximal value in the biased function is larger, the injurer would still invest more when the distribution is biased in that region.
probability of fault will influence the injurer’s incentives to invest in care.

The court will find the injurer liable for the accident whenever the observed level of care falls below \( \ln \left( \frac{H^{1+\sigma} + H^{1-\sigma}}{2} \right) \). Notice that the value is higher than the standard of care, \( \ln x \), for \( \sigma > 0 \). Knowing this, the injurers cost function is:

\[
(13) \quad C_{\text{biased}}^{\text{PE}}(p(x)=e^{-x}) = \begin{cases} 
 x + x^{-\alpha}H \\ -\sigma \ln(H) > x - \ln \left( \frac{H^{1+\sigma} + H^{1-\sigma}}{2} \right) \\
 x + \left( \ln \left( \frac{H^{\sigma+1} + H^{1-\sigma}}{2} \right) - x + \sigma \ln(H) \right) e^{-x}H \\
 \sigma \ln(H) \geq x - \ln \left( \frac{H^{\sigma+1} + H^{1-\sigma}}{2} \right) \geq \sigma \ln(H) \\
 x - \ln \left( \frac{H^{\sigma+1} + H^{1-\sigma}}{2} \right) > \sigma \ln(H)
\end{cases}
\]

Solving for the first order condition, for the middle region, we obtain:

\[
(14) \quad 1 = \frac{H^{(1-x+\sigma \ln(H))}+\ln \left( \frac{H^{\sigma+1} + H^{1-\sigma}}{2} \right)}{2 \sigma \ln(H)e^{x}}
\]

Fig. 4 shows the injurer’s investment in care as a function of the uncertainty level. The lower curve (dashed) shows the injurer’s investment in care, assuming the court does not consider the occurrence of the accident as part of the evidence (symmetric error), i.e. the level of \( x \) that satisfies Exp.12. The higher curve (solid) shows the injurer’s investment in care assuming the court updates the probability of fault based rational hindsight, i.e. the level of \( x \) that satisfies Exp.14.
As can be seen in fig. 14, when the court updates the probability of negligence, the injurer’s incentives are improved. The result should not come as a surprise. When precaution measures are extremely effective, the court will almost always find the injurer liable. Knowing this, the injurer fully internalizes the costs of accident, which makes her invest optimally in care. To put in different words, when precaution measures are very effective, and the court takes this into account when evaluating the probability of fault, the liability regime becomes closer to strict liability than negligence.

3. DISCUSSION

The model reveals some key features of tort liability when the detection of the injurer’s behavior in care is imprecise. The effectiveness of available precaution measures plays a key role, for two reasons – First, as the precaution technology becomes more effective, the residual expected harm from the activity, after investing appropriately in care, gets smaller. As a result, the injurer is less affected from being found liable by mistake, and has less of a motivation to overinvest in care to escape liability. That means that in equilibrium negligence regime would cause injurers to underinvest in care when precaution measures are more effective, and to overinvest in care when precaution measures are less effective. Second, updating the probability of fault based on the occurrence of an accident has a week effect when precaution measures are less effective, and potentially a very strong effect when precaution measures are more effective.

These insights offer a now way to examine some familiar tort doctrines. In the rest of the Paper I will try to show how these results might explain the evidentiary rule prohibiting the plaintiff from presenting evidence about subsequent precaution measures that the defendant took; Strict liability for abnormally dangerous activities and shifting the burden of proof under the doctrine of res ipsa loquitur.

3.1. Prohibiting Relevant Evidence

Article IV of the Federal Rules of Evidence prohibits the use of various relevant evidence, mainly on social policy grounds. For example, Rule 411 determines that the plaintiff cannot bring evidence showing that the defendant was insured against liability to prove that the defendant acted negligently. This evidence might have some probative value,

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17 In terms of the two effects of uncertainty – the partial internalization effect does not create underdeterrence since the injurer internalizes the entire cost of accident. Similarly, there is no private benefit from investing in care since the injurer knows that extra investment will not substantially reduce her expected liability.
but allowing it would discourage potential injurers from purchasing liability insurance, which is unwarranted.

Similarly, Rule 407 states that "When measures are taken that would have made an earlier injury or harm less likely to occur, evidence of the subsequent measures is not admissible to prove negligence...". Again, this rule can be explained on policy grounds – while the evidence might be probative, it would discourage the defendant from investing in new precaution measures after the accident, fearing she might be found liable as a result.

However, (Posner 1999) suggested that Rule 407 may be designed also to combat hindsight bias, but rejected this notion, stating that "hindsight bias is often rational (for example, when the occurrence of an accident shows that a hypothetical possibility was a real one) and thus not an illusion at all". It appears that Judge Posner rejected the role that Rule 407 might play in combating hindsight, since irrational hindsight is "limited and weak", and rational hindsight is beneficial. The analysis above only considered rational hindsight, and showed that it is often problematic. If Rule 407 reduces the inference that can be made from the accident, it might be beneficial, even if the hindsight is rational.

Arguably, we could have adopted a broader rule, prohibiting the plaintiff from presenting evidence regarding the occurrence of an accident. However, this is impractical. A prerequisite for a case in tort is that the plaintiff suffered harm because of an accident. Even if the plaintiff cannot mention the occurrence of an accident, the judge and jury all know that an accident has occurred.

3.2. Abnormally Dangerous Activities

The doctrine of abnormally dangerous activities places strict liability on the injurer whenever the injurer's activity is deemed abnormally dangerous. Restatement 3d of Torts: Liability for physical and Emotional Harm (2010) states at § 20 that "An activity is abnormally dangerous if: (1) The activity creates a foreseeable and highly significant risk of physical harm even when reasonable care is exercised by all actors;"

Reporters of the restatement thought that the basis of the doctrine is to expand liability, and offer the plaintiff a way to claim that the entire activity causes more harm than benefit, and thus should be deemed negligent. Such a claim might be difficult to prove
under regular negligence, and strict liability in these cases might solve the problem.\textsuperscript{18}

The model here offers a different explanation to the doctrine. Notice that for the definition to apply the risk must be substantial, and remain so even when adequate care is taken. In the terms of the model, that means that prevention technology is not very effective. Recall that when the technology is effective, the residual harm after investing adequately in care must be small, even if potential harm is high. In fact, that is why uncertainty may cause underdeterrence. When residual harm is high, uncertainty would usually cause overdeterrence. Thus, if the purpose of the rule was to encourage the injurer to invest \textit{more} in care, it seems that no change was needed. Furthermore, if the court applies rational hindsight (or irrational for that matter) overdeterrence becomes more severe.

However, if the purpose of the doctrine is to encourage the injurer to incest \textit{less} in care, it makes perfect sense. Under strict liability the injurer invests the optimal amount in care. Knowing that she cannot influence the expected liability by investing more in care eliminates the private benefit from added precaution.

\textbf{3.3. Res Ipsa Loquitur}

Last, Consider the doctrine of res ipsa loquitur. Restatement 3d of Torts: Liability for physical and Emotional Harm (2010) states at § 17 that “The factfinder may infer that the defendant has been negligent when the accident causing the plaintiff's harm is a type of accident that ordinarily happens as a result of the negligence of a class of actors of which the defendant is the relevant member.”

The restatement offers the next example as an illustration of the doctrine.

\textit{a car driven by the defendant runs off the road, injuring a pedestrian. In considering this category of accidents--cars that run off the road--several possible causes can be identified, including motorist negligence; some mechanical problem with the car; some defect in the roadway; and very adverse weather conditions. If the jury can reasonably believe that motorist negligence is most often the cause when cars run off the road, then, absent further evidence about the particular incident, the jury can reason from the general to the particular and hence properly infer that the defendant motorist

\textsuperscript{18}Restatement 3d of Torts: Liability for physical and Emotional Harm, § 20 cmt. b.
was probably negligent."

Notice that this formulation of the rule is a good example for rational hindsight – the jury is asked to infer, in case of uncertainty regarding the injurer's conduct, if the injurer invested adequately in care, based on the probability of an accident occurring given negligent investment and proper investment in care.

If, for the doctrine to apply, the plaintiff must show that rational hindsight substantially influences the prior estimation of fault, the doctrine is efficient. Remember that when the court can make strong inference from the accident, it means that precaution measures are more effective, and uncertainty is more likely to cause underdeterrence. Furthermore, the doctrine applies when the plaintiff cannot present evidence regarding defendant's behavior. In such cases the level of uncertainty is high, again suggesting underdeterrence. We have seen that in these cases we want the court to apply rational hindsight, which is exactly what the doctrine states.

References


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